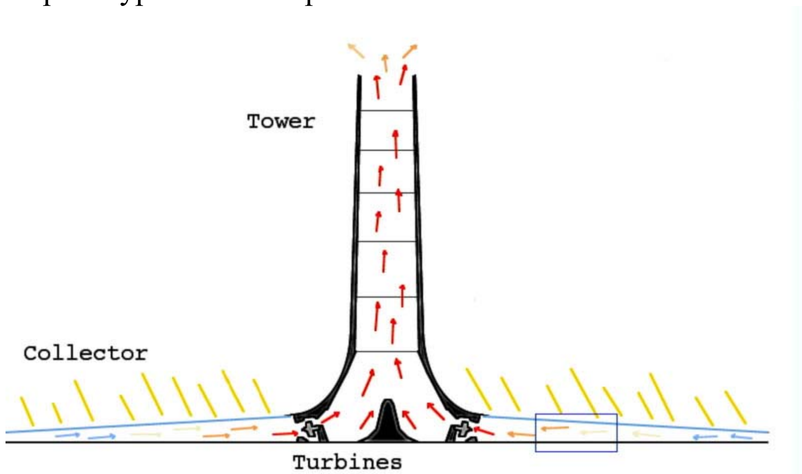
Homework # 1

The Solar Chimney

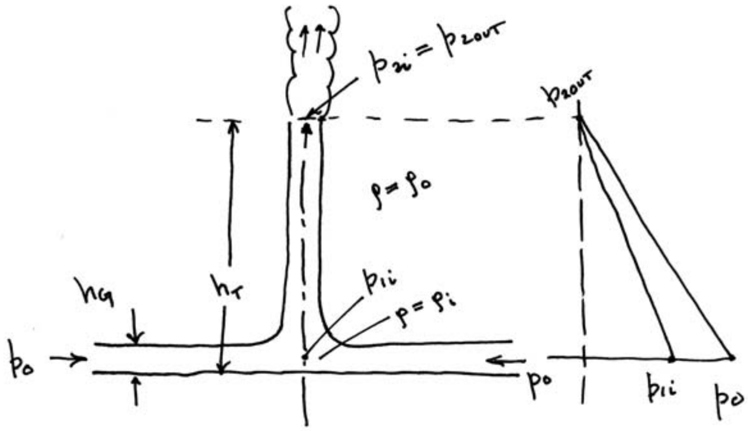
One possible scheme for generating electrical power is by means of a solar chimney, or solar tower. A schematic of the tower is shown below (from Wikipedia), and a photograph of a prototype tower in Spain is also shown.



http://en.wikipedia.org/wiki/Solar_updraft_tower

http://www.enviromission.com.au/project/prototype.htm

Air is heated near the ground by trapping solar radiation in a flat, circular glass-roofed greenhouse. The heated air rises in the tower, and the updraft is used to drive a turbine (or several turbines). That this might work can be seen in the sketch below. Because the hot air within the tower weighs less than the air outside the tower, one sees that there is a lower pressure at the center within the tower (p_{1i}) than at ground level outside the tower (p_o). Based upon an overly simple hydrostatic model, the hot air flows inwards, acquires energy, and continues up the tower.



$$p_o = p_2 + \rho_o g h_T$$
$$p_{1i} = p_{2i} + \rho_i g h_T \equiv p_2 + \rho_i g h_T$$
$$p_o - p_{1i} = (\rho_o - \rho_i) g h_T \equiv \Delta\rho g h_T$$

If we want quantitative information—and we always do—we must improve the model. The assumptions are that:

density $\rho=\rho_i$ everywhere inside the greenhouse and tower;
density $\rho=\rho_o$ everywhere outside the device;
cross-sectional area of the tower $=A_2$ is constant;
ignore the details of heat transfer from the sun. Simply assume
that the function of the sun/greenhouse is to bring the air to density to $\rho=\rho_i$.

The energy flux across any cross-section of the tower is $\frac{1}{2}\rho_i U_2{}^3 A_2$. The maximum rate that energy can be extracted from the flow in the tower is this quantity. Thus

$$(Power)_{\max} = \frac{1}{2}\rho_i U_2{}^3 A_2$$

Model I: Assume no losses in the system, and use Bernoulli's equation to estimate the flow velocity within the tower, and hence to estimate the power out. Since this estimate does not account for losses, the estimate will be extravagantly optimistic.

Model II: Include the turbine(s)—the driving pressure drop across the turbine(s)—and the losses in the rest of the system. Specifically, for the sketch below, let

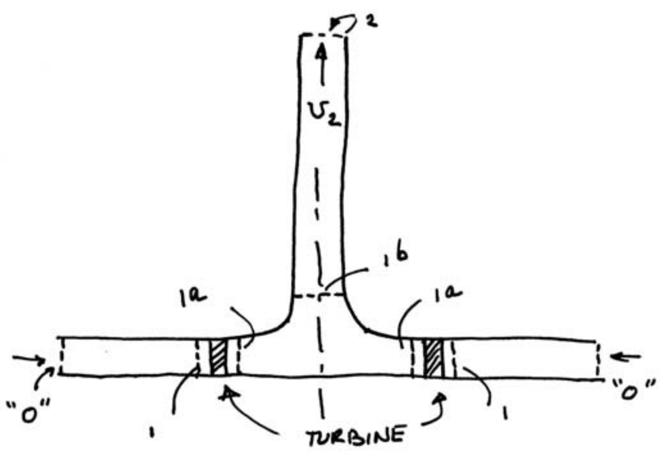
$$p_1 - p_{1a} = K_{Turbine} \frac{1}{2} \rho_i U_1^{\ 2}$$, for the turbine pressure coefficient,

and—for the duct loss at the base of the tower—let

$$p_{1a} - p_{1b} = K_{Duct} \frac{1}{2} \rho_i U_2^{\ 2}.$$

Use Bernoulli's equation—including the loss terms—to add up all the pressure drops in the system from "o" to "2". Equate this *required* pressure drop to the *available* pressure drop,

$$p_o - p_2 = \rho_o g h_T .$$



The power extracted by the turbine will then be

$$Power = (p_1 - p_{1a}) A_{1a} U_{1a} = K_T \frac{1}{2} \rho_i U_{1a}^{\ 3} A_{1a} .$$

Questions:

1. Write an expression for the maximum power output using Model I. Take the design numbers for the proposed Australian tower to be: $h_T = 1000$ m, for the central tower, D = 150 m, $T_o = 290$ K, $T_i = 320$ K, and let $A_o >> A_2$. Compute an estimate for the velocity U_2, for the mass flow, and for the maximum possible power out, P_{max}. Comment.

2. Write an expression for the power output using Model II. Take the turbine pressure coefficient to be $K_T = 0.85$, and the duct losses as $K_D = 0.6$. Let $A_1 = A_{1a}$, and let $A_{1b} = A_2$. Compare the final expression with the previous expression derived in 1. Comment. For the particular case $A_1 = A_2$, estimate the velocity $U_1 = U_{1a} = U_2$, the mass flow, and the power out.

3. Part of the design problem might be to site the turbine in the an advantageous position in the duct. Discuss how you might determine the most advantageous position—in terms of the cross-sectional area—for the turbine site. That is, let A_1 be different from A_2 and choose it most advantageously.